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LETTER TO THE EDITOR

Quenched impurities and tricriticality in classical and quantum systems

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Abstract. A generalised model, intended to describe a number of quantum systems in the presence of short-range correlated quenched impurities, is studied by the renormalisation group approach. To first order in ε, it is shown that the tricritical behaviour is unstable towards impurity perturbations for both finite and zero temperatures. A brief discussion of the zero-temperature tricriticality in pure systems is also presented.

The influence of short-range correlated quenched impurities on the static critical behaviour of n-vector classical systems is now well established (Lubensky 1975, Khmel'nitskii 1975, Grinstein and Luther 1976, Ma 1976, Shalaev 1977). The dynamics of these impure systems have been investigated by Grinstein *et al* (1977). More recently, the effects of extended (Dorogovstsev 1980a, b, Boyanovsky and Cardy 1982, Prudnikov 1983) and long-range correlated (Weinrib and Halperin 1983) impurities have also been explored. Furthermore, a study of the impurity influence on critical behaviour of quantum systems has been made in the short-range (Busiello *et al* 1984a, Uzunov 1984) and long-range (Busiello *et al* 1984b) cases respectively. Note that there are not, up to now, investigations of impurity influence on systems with tricritical points.

The present letter is devoted to a renormalisation group (RG) study of this problem. Instead of limiting ourselves to classical systems (Wegner and Riedel 1973, Stephen and McCauley 1973, Nelson and Fisher 1975, Rudnick and Nelson 1976), we shall generalise the discussion starting from a functional model which describes a number of quantum systems (see e.g. Hertz 1976, Gerber and Beck 1977, Morf *et al* 1977, Busiello *et al* 1983). In this way, we shall have the possibility to explore the impurity effects on zero-temperature quantum tricriticality too. Since the pure tricritical behaviour in the quantum regime has not been investigated up to now, a brief discussion of this subject is also included.

The generalised d-dimensional quantum model we consider is described by the functional

$$\mathcal{H}\{\psi,\varphi\}=\sum_{\alpha,q}(r+ck^{\sigma}+g(q))|\psi_\alpha(q)|^2+\frac{1}{V^{1/2}}\sum_{\alpha,k_1k_2;\omega_l}\varphi(\boldsymbol{k}_1-\boldsymbol{k}_2)\psi^*_\alpha(\boldsymbol{k}_1,\omega_l)\psi_\alpha(\boldsymbol{k}_2,\omega_l)$$
$$+\frac{u_4T}{4V}\sum_{\alpha,\beta;q_1q_2q_3}\psi^*_\alpha(q_1)\psi^*_\beta(q_2)\psi_\alpha(q_3)\psi_\beta(q_1+q_2-q_3)$$

$$+\frac{u_6 T^2}{6V^2}\sum_{\alpha,\beta,\gamma;q_1\ldots q_5}\psi^*_\alpha(q_1)\psi^*_\beta(q_2)\psi^*_\gamma(q_3)\psi_\alpha(q_4)\psi_\beta(q_5)\psi_\gamma(q_1+q_2+q_3-q_4-q_5) \tag{1}$$

where $q\equiv(\boldsymbol{k},\omega_l)$, $\boldsymbol{k}$ is the wavevector with a cut-off, $\omega_l=2\pi lT$ ($K_B=\hbar=1$, $l=0,\pm1,\ldots$), T is the temperature, $\psi(q)\equiv\{\psi_\alpha(q);\alpha=1,\ldots,n/2;n\geqslant2\}$ is a complex $(n/2)$-component field and V is the volume. In (1), $0<\sigma\leqslant2$, $\varphi(\boldsymbol{k})$ is a random function describing the impurities and the function $g(q)$ and the definition of the parameters r, c, u_4, u_6 depend on the particular model under study. Here, it is assumed that $\varphi(\boldsymbol{k})$ is governed by a Gaussian distribution with average

$$[\varphi(\boldsymbol{k})\varphi(\boldsymbol{k}')]_{\rm av}=\Delta\delta_{\boldsymbol{k},-\boldsymbol{k}'},\qquad \Delta\geqslant0. \tag{2}$$

For the function $g(q)$ we consider the form

$$g(q)=f(\omega_l)/k^{m'},\qquad f(\omega_l)=\begin{cases}-\mathrm{i}\omega_l,\\ |\omega_l|^m,\end{cases} \tag{3}$$

where $m'\geqslant0$ and $m\geqslant1$. The expression (3) is quite general and describes several quantum systems: (i) the XY model (Gerber and Beck 1977, Kopec and Kozlowski 1983) and the Bose gas (Busiello and De Cesare 1980, Uzunov 1981, Walasek 1983) for $m'=0$ and $f(\omega_l)=-\mathrm{i}\omega_l$; (ii) structural phase transitions (Morf *et al* 1977, Millev and Uzunov 1983) for $m'=0$ and $m=2$; (iii) superconductors (Uzunov 1980) for $m'=0$ and $m=1$; (iv) itinerant magnets (Hertz 1976); (v) exciton phase transitions (Baba *et al* 1979) for $m'=0$ and $m=1$ or 2.

Our investigation is conveniently performed in the framework of the differential RG equations (Wegner and Houghton 1973, Rudnick and Nelson 1976) without using the replica trick. In the usual RG transformation, the rescaling factors are $b=\mathrm{e}^l$ ($l\geqslant0$) for the wavevectors and $\xi_l=\mathrm{e}^{l(1-\eta/2)}$ for the order parameter, where η is the Fisher exponent. Here, we do not find it necessary to rescale the frequencies and we assume units in which $\Lambda=c=1$. Since, in any case, there are no k^σ-type contributions to first order in $\varepsilon=d_{\rm cu}-d$, where $d_{\rm cu}$ is the upper critical space dimensionality, we obtain $\eta=2-\sigma$. Furthermore, the following recursion relation for the temperature (or $f(\omega_l)$) is found:

$$\mathrm{d}T/\mathrm{d}l=z(\Delta)T \tag{4}$$

where

$$z(\Delta)=\frac{\sigma+m'}{m}+\frac{1}{m}\delta_{m',0}K_d\frac{\Delta}{(1+r)^2} \tag{5}$$

and $K_d=2^{1-d}\pi^{-d/2}/\Gamma(d/2)$. As we see, an $\mathrm{O}(\varepsilon)$ contribution to the dynamical critical exponent $z=z(\Delta^*)$ at a random fixed point (RFP) is possible only if $m'=0$. If $m'\neq0$, ε-corrections to z are expected to second order in ε. Formally (5) is also true for case (i) with $m=1$. The RG equations for the parameters r, $v=u_4T$, $w=u_6T^2$ and Δ are

$$\begin{aligned}
\mathrm{d}r/\mathrm{d}l&=\sigma r+\tfrac{1}{4}(n+2)K_dS_1(r,T)v-K_d\Delta/(1+r),\\
\mathrm{d}v/\mathrm{d}l&=(2\sigma-d)v-\tfrac{1}{4}K_d[(n+6)S_2(r,T)+2\tilde{S}_2(r,T)]v^2+(n+4)K_dS_1(r,T)w\\
&\qquad+6K_dv\Delta/(1+r)^2,\\
\mathrm{d}w/\mathrm{d}l&=(3\sigma-2d)w-\tfrac{3}{4}K_d[(n+10)S_2(r,T)+4\tilde{S}_2(r,T)]vw+15K_dw\Delta/(1+r)^2,\\
\mathrm{d}\Delta/\mathrm{d}l&=(2\sigma-d)\Delta+4K_d\Delta^2/(1+r)^2-\tfrac{1}{2}(n+2)K_dS_2(r,T)v\Delta,
\end{aligned} \tag{6}$$

where

$$S_1(r,T)=\lim_{\tau\to0^+}\sum_{\omega_l}\exp(\mathrm{i}\omega_l\tau)G_0(1,\omega_l),\qquad S_2(r,T)=\sum_{\omega_l}G_0^2(1,\omega_l),$$
$$\tilde{S}_2(r,T)=\sum_{\omega_l}|G_0(1,\omega_l)|^2. \tag{7}$$

In (7), $G_0=[r+k^{\sigma}+g(q)]^{-1}$ and the summation with $\tau\to0^+$ is necessary only when $f(\omega_l)=-\mathrm{i}\omega_l$.

Now we perform the analysis of (4)–(6) for the two different regimes $T\neq0$ and $T=0$. First we consider the finite-temperature classical behaviour. In this case the Matsubara frequencies are irrelevant for the static scaling behaviour and we can neglect (4), (5) as well as the frequencies $\omega_l\neq0$ in (6). Thus, for any model, one obtains

$$\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}l}&=\sigma r+\frac{n+2}{4}K_d\frac{v}{1+r}-K_d\frac{\Delta}{1+r},\\
\frac{\mathrm{d}v}{\mathrm{d}l}&=(2\sigma-d)v-\frac{n+8}{4}K_d\frac{v^2}{(1+r)^2}+(n+4)K_d\frac{w}{1+r}+6K_d\frac{v\Delta}{(1+r)^2},\\
\frac{\mathrm{d}w}{\mathrm{d}l}&=(3\sigma-2d)w-\tfrac{3}{4}(n+14)K_d\frac{vw}{(1+r)^2}+15K_d\frac{w\Delta}{(1+r)^2},\\
\frac{\mathrm{d}\Delta}{\mathrm{d}l}&=(2\sigma-d)\Delta+4K_d\frac{\Delta^2}{(1+r)^2}-\frac{n+2}{2}K_d\frac{v\Delta}{(1+r)^2}.
\end{aligned}\tag{8}$$

These equations are a direct generalisation of those presented in previous papers (Rudnick and Nelson 1976, Blankschtein and Aharony 1983) for pure classical systems. For a discussion of the tricritical or critical behaviour, as usual, we determine the fixed points of (8) and the exponents λ_i $(i=r,v,w,\Delta)$ which enter the linearised version of (8) around each fixed point (FP).

A Gaussian fixed point (GFP) $(r^*=v^*=w^*=\Delta^*=0)$ always exists and the corresponding exponents $\{\lambda_i^{(G)}\}$ are $\lambda_r^{(G)}=\sigma$, $\lambda_v^{(G)}=\lambda_\Delta^{(G)}=2\sigma-d$ and $\lambda_w^{(G)}=3\sigma-2d$. It is stable for $d>2\sigma$, describing a Gaussian critical behaviour, and doubly unstable towards both v and Δ perturbations for $\frac{3}{2}\sigma\leqslant d<2\sigma$. Since in the pure case the GFP is assumed to describe a tricritical behaviour for $\frac{3}{2}\sigma\leqslant d<2\sigma$, the last result indicates an instability of tricriticality towards the randomness. Here we shall not be concerned with the region $d<\frac{3}{2}\sigma$ where non-Gaussian tricritical exponents appear for pure classical systems (Stephen and McCauley 1973). By using $\varepsilon=2\sigma-d$ as expansion parameter the usual pure FP and the RFP (Lubensky 1975, Ma 1976), with the additional coordinate $w^*=0$, appear which are both stable towards w perturbation since $\lambda_w^{(P)}=-\sigma-[(n+26)/(n+8)]\varepsilon<0$ and $\lambda_w^{(R)}=-\sigma-5(n+8)/8(n-1)<0$. They govern the critical pure and random behaviour for $n>4$ and $2\leqslant n<4$ respectively. When the RFP is stable, equation (5) for $m'=0$ gives the dynamical critical exponent $z=\sigma/m+(1/m)[(4-n)/8(n-1)]\varepsilon$ $(m\geqslant1)$.

We now consider the zero-temperature quantum behaviour. The appropriate RG equations for the parameters r, u_4, u_6, Δ are obtained from (6) taking into account (5) and then setting $T=0$. They are

$$\begin{aligned}
\mathrm{d}r/\mathrm{d}l&=\sigma r+\tfrac{1}{4}(n+2)K_dF_1(r,0)u_4-K_d\Delta/(1+r),\\
\mathrm{d}u_4/\mathrm{d}l&=(2\sigma-d-z_0)u_4-\tfrac{1}{4}K_d[(n+6)F_2(r,0)+2\tilde{F}_2(r,0)]u_4^2\\
&\quad+(6-\delta_{m',0}/m)K_du_4\Delta/(1+r)^2,
\end{aligned}$$

$$\mathrm{d}u_6/\mathrm{d}l = (3\sigma - 2d - 2z_0)u_6 - \tfrac{3}{4}K_d[(n+10)F_2(r,0) + 4\tilde{F}_2(r,0)]u_4u_6 + (15 - 2\delta_{m,0}/m)K_d u_6\Delta/(1+r)^2, \tag{9}$$

$$\mathrm{d}\Delta/\mathrm{d}l = (2\sigma - d)\Delta + 4K_d\Delta^2/(1+r)^2 - \tfrac{1}{2}(n+2)K_dF_2(r,0)u_4\Delta,$$

where $z_0 = (\sigma + m')/m$ and $F_i(r, T) = TS_i(r, T)$ $(i = 1, 2)$, $\tilde{F}_2(r, T) = T\tilde{S}_2(r, T)$. The functions $F_i(r, 0)$ and $\tilde{F}_2(r, 0)$ are different for different models. Here, we shall discuss only two models so that the main features of the $(T = 0)$ tricritical behaviour will be clarified.

(*a*) *Non-ideal Bose gas*

The $(T = 0)$ RG equations are obtained from (9) with $F_i(r, 0) = 0$ $(i = 1, 2)$, $\tilde{F}_2(r, 0) = \frac{1}{2}(1+r)^{-1}$, $m = 1$ and $m' = 0$ $(z_0 = \sigma)$. Firstly, we briefly discuss the pure tricritical behaviour, setting in (9) $\Delta \equiv 0$ so that $z = \sigma + \mathrm{O}(\varepsilon^2)$. In this situation, a GFP $(r^* = u_4^* = u_6^* = 0)$ always exists with exponents $\lambda_r^{(G)} = \sigma$, $\lambda_{u_4}^{(G)} = \sigma - d$ and $\lambda_{u_6}^{(G)} = \sigma - 2d$. It is stable for $d > \sigma$, describing a Gaussian critical behaviour, and unstable towards the parameter u_4 for $\frac{1}{2}\sigma \leqslant d < \sigma$ (the region $d < \frac{1}{2}\sigma$ is not of real interest as $0 < \sigma/2 \leqslant 1$). Thus the GFP describes the tricriticality for $\frac{1}{2}\sigma \leqslant d < \sigma$ and a dimensional crossover $d \to d + \sigma$ takes place for $T \to 0$ as obtained before for criticality (Hertz 1976, Busiello *et al* 1983). If we assume $\varepsilon = \sigma - d$ as expansion parameter, the non-trivial FP $(r^* = 0,\ u_4^* = (4/K_d)\varepsilon,\ u_6^* = 0)$ appears with $\lambda_r = \sigma$, $\lambda_{u_4} = -\varepsilon$, $\lambda_{u_6} = 2\varepsilon - \sigma < 0$, which describes a pseudogaussian transition for $d < \sigma$ (De Cesare 1978, Busiello and De Cesare 1980, Uzunov 1981). Of course a critical–tricritical crossover occurs in the domain $\frac{1}{2}\sigma < d < \sigma$ (Blankschtein and Aharony 1983).

Now we are in a position to investigate the effect of impurities $(\Delta \neq 0)$. The fourth of equations (9) gives two FP values $\Delta^* = 0$ and $\Delta^* = -\varepsilon/4K_d$ with $\varepsilon = 2\sigma - d$. The latter corresponds to a RFP which is unphysical $(\Delta^* < 0)$ for $d < 2\sigma$ and physical but unstable $(\lambda_\Delta^{(R)} = -\varepsilon > 0)$ for $d > 2\sigma$. Thus, the only type of FP we have is characterised by the value $\Delta^* = 0$. This is in complete contrast with the $(T \neq 0)$ case where one obtains a value $\Delta^* = \mathrm{O}(\varepsilon)$ and hence an ε correction to the dynamic exponent $z = z(\Delta^*)$. It is quite easy to see from (9) that the possible pure FPs with $\Delta^* = 0$, intended to describe either criticality or tricriticality, are unstable towards Δ perturbation for $d < 2\sigma$. For $d > 2\sigma$, the behaviour of the random system is governed by the stable GFP $(\lambda_\Delta^{(G)} = 2\sigma - d < 0)$ and we would have a Gaussian transition with $\nu = \lambda_r^{-1} = 1/\sigma$, $\eta = 2 - \sigma$, $z = \sigma$. These results for criticality have already been derived previously (Busiello *et al* 1984a, Uzunov 1984).

(*b*) *Structural phase transitions*

The appropriate RG equations can be obtained by setting in equations (9) $m' = 0$, $m = 2$ $(z_0 = \sigma/2)$, $F_1(r, 0) = \frac{1}{2}(1+r)^{-1/2}$ and $F_2(r, 0) = \tilde{F}_2(r, 0) = \frac{1}{4}(1+r)^{-3/2}$. In contrast with the bosonic case, they contain the order parameter dimensionality n. The pure tricriticality $(\Delta \equiv 0)$ can be discussed as for the Bose system. The GFP with exponents $\lambda_r^{(G)} = \sigma$, $\lambda_{u_4}^{(G)} = \frac{3}{2}\sigma - d$, $\lambda_{u_6}^{(G)} = 2(\sigma - d)$ is unstable towards u_4 for $\sigma \leqslant d < \frac{3}{2}\sigma$ and describes the tricriticality. The usual criticality is governed by the FP $(r^* = -(2/\sigma)[(n+2)/(n+8)]\varepsilon$, $u_4^* = [16/K_d(n+8)]\varepsilon$, $u_6^* = 0)$ where $\varepsilon = \frac{3}{2}\sigma - d$, stable for $d < \frac{3}{2}\sigma$ and by the GFP stable for $d > \frac{3}{2}\sigma$. Also in this case a dimensional crossover $d \to d + \sigma/2$ occurs for $T \to 0$.

When disorder is present, as for the Bose gas, the only type of physical FP is characterised by the value $\Delta^* = 0$. Also here we find that the pure critical and tricritical behaviours are unstable towards impurities for $d < 2\sigma$ and a Gaussian behaviour occurs for $d > 2\sigma$.

The previous $(T = 0)$ results are true also for other quantum models with $z = (\sigma + m')/m + \mathrm{O}(\varepsilon^2)$. In general, as for criticality, the tricritical behaviour for pure systems obeys a dimensional crossover $d \to d+z$ and is unstable towards an appearance of impurities for any n and $d < 2\sigma$.

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References

Baba Y, Nagai T and Kawasaki K 1979 *J. Low Temp. Phys.* **36** 1
Blankschtein D and Aharony A 1983 *Phys. Rev.* B **28** 386
Boyanovsky D and Cardy J L 1982 *Phys. Rev.* B **26** 154
Busiello G and De Cesare L 1980 *Nuovo Cimento* B **59** 327
Busiello G, De Cesare L and Rabuffo I 1983 *Physica* A **117** 445
—— 1984a *Phys. Rev.* B in press
—— 1984b *Phys. Lett.* A in press
De Cesare L 1978 *Lett. Nuovo Cimento* **22** 325, 632
Dorogovtsev S N 1980a *Sov. Phys.–Solid State* **22** 188
—— 1980b *Sov. Phys.–Solid State* **22** 3658
Fisher M E, Ma K S and Nickel B 1982 *Phys. Rev. Lett.* **29** 917
Gerber P R and Beck H 1977 *J. Phys. C: Solid State Phys.* **10** 4013
Grinstein G and Luther A 1976 *Phys. Rev.* B **13** 1329
Grinstein G, Ma K S and Mazenko G R 1977 *Phys. Rev.* B **15** 258
Hertz J A 1976 *Phys. Rev.* B **14** 1165
Khmel'nitskii D E 1975 *Sov. Phys.–JETP* **41** 981
Kopec T K and Kozlowski G 1983 *Phys. Lett.* A **95** 104
Lubensky T C 1975 *Phys. Rev.* B **11** 3573
Ma K S 1976 *Modern theory of critical phenomena* (London: Benjamin)
Millev Y T and Uzunov D I 1983 *J. Phys. C: Solid State Phys.* **16** 4107
Morf R, Schneider T and Stoll E 1977 *Phys. Rev.* B **16** 462
Nelson D R and Fisher M E 1975 *Phys. Rev.* B **11** 1030
Prudnikov V V 1983 *J. Phys. C: Solid State Phys.* **16** 3685
Rudnick J and Nelson D R 1976 *Phys. Rev.* B **13** 2208
Shalaev B I 1977 *Sov. Phys.–JETP* **46** 1204
Stephen M J and McCauley J L Jr 1973 *Phys. Lett.* **44A** 89
Uzunov D I 1980 *Phys. Lett.* **78A** 395
—— 1981 *Phys. Lett.* **87A** 11
—— 1984 Unpublished
Walasek K 1983 *Phys. Lett.* **98A** 346
Wegner F J and Houghton A 1983 *Phys. Rev.* A **8** 401
Wegner F J and Riedel E K 1973 *Phys. Rev.* B **7** 248
Weinrib A and Halperin B I 1983 *Phys. Rev.* B **27** 413